Subjective Evaluation

Class 8
How to Evaluate Students?

• Essay vs. multiple choice
• Oral exam vs. written exam
• Participation vs. attendance
Review:
Optimal Contract with Hidden Action

Multiple Tasks
\[ b = \frac{1}{1 + r(\theta_1 + \theta_2)} \]

Risk-averse agent
\[ b = \frac{1}{1 + r\theta} \]

Multiple Signals
\[ b = \frac{1}{1 + r(\theta - \rho^2)} \]

Risk-neutral agent
\[ b = 1 \]

Low-power incentives

High-power incentives
Pitfalls of Tying Pay to Outcomes

- $\frac{\partial w}{\partial q} = b \approx 0$ in many occupations
- $b = \frac{1}{1+r(\theta-\rho^2)}$
  - Agent extremely risk averse ($r \to \infty$)
  - Lack of control over output ($\theta \to \infty$)
  - Lack of good signals of performance ($\rho \to 0$)

- Other Explanations:
  - Multiple tasks
  - Imperfect performance measures (today)
  - Non-financial incentives
Imperfect Performance Measures

- Two main problems with measuring agent’s performance:

  1. Gaming
     - The outcome cannot be measured or verified, and
     - The agent can manipulate alternative performance measures

  2. Biased rating
     - The outcome is measured by a third party that lacks interest or ability to report the outcome accurately
Objectives for Today

1. Examples of Gaming
2. Consequences of Gaming
3. Subjective Evaluation as Solution to Gaming Problem
4. Biased Rating in Multilayer Organizations
Performance Measures

- **Objective**: can be observed and verified
- **Subjective**: can be observed, but not verified

- Often, objective ≠ subjective

- Contracts based on imperfect objective measures may result in dysfunctional behaviour
  
  *(You get what you pay for, not always what you want!)*
Example 1: H. J. Heinz Company

- Division managers received bonuses only if earnings increased from the prior year

⇒ The managers delivered consistent earnings growth by manipulating the timing of shipments to customers and by prepaying for services not yet received
Example 2: Dun & Bradstreet

- Salespeople earned no commission unless the customer bought a larger subscription to the firm's credit-report services than in the previous year.

⇒ In 1989 the company faced millions of dollars in lawsuits following charges that its salespeople deceived customers into buying larger subscriptions by fraudulently overstating their historical usage.
Example 3: Sears

- Pay mechanics in auto-repair shops based on the profits from repairs authorized by customers

⇒ Mechanics misled customers into authorizing unnecessary repairs, leading California officials to prepare to close Sears' auto-repair business statewide
<table>
<thead>
<tr>
<th>Element</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Parties</td>
<td>Principal, Agent</td>
</tr>
<tr>
<td>Production Technology</td>
<td>( q = e + u ), where ( u \sim (0, \theta) )</td>
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<tr>
<td>Information</td>
<td>( e ) cannot be observed or verified</td>
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<tr>
<td></td>
<td>( q ) can be observed but not verified (subjective)</td>
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<tr>
<td></td>
<td>( p ) can be observed and verified (objective)</td>
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<tr>
<td>Contract</td>
<td>((a, b)) where ( w = a + bp )</td>
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<tr>
<td>Payoffs</td>
<td>Agent: ( U = u(w) - c(e) ), with ( r = 0 ) and ( c(e) = 0.5e^2 )</td>
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<td></td>
<td>Principal: ( V = V(q - w) ), with ( s = 0 )</td>
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<tr>
<td>Outside Options</td>
<td>( R = 0 = S )</td>
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<td>Timing</td>
<td>1. P design contract ((a,b))</td>
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<td>2. A accepts or rejects</td>
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<td>3. If A accepts, A chooses ( e )</td>
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<td>4. Production and payoffs</td>
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Performance Measures

- q can be observed, but not verified (subjective)
- p can be observed and verified (objective)
- p is an imperfect measure of q:
  - p=kq
  - Only the agent knows k (private information)
  - Principal knows E[k]=1, Var[k]=θ

- E[p]=ke for the agent
- E[p]=e for the principal
Constraints

\[ E[U] = E[w] - c(e) = a + bE[p] - 0.5e^2 \]
\[ = a + bke - 0.5e^2 \]

**Incentive Compatibility Constraint**
- Max \( E[U] \)
- \( \Rightarrow (IC) \ bk - e = 0 \Rightarrow \underline{\text{______________}} \)

**Participation Constraint**
- \( E[U] = E[w] - c(e) \geq R = 0 \)
- \( \Rightarrow E[w] = \underline{\text{__________________________}} \)
Recall that k is the agent’s private information. Therefore, the principal must use the expected value of e=bk.

- $E[V] = E[q] - E[w]$
- $= E[e] - E[0.5e^2]$  \hspace{1cm} (PC)
- $= E[bk] - E[0.5(bk)^2]$  \hspace{1cm} (IC)
- $= b - 0.5b^2E[k^2]$
- $= b - 0.5b^2(1 + \theta) \hspace{1cm} E[k^2] = \theta + [E(k)]^2$
Optimal Contract

- $E[V] = b - 0.5b^2(1+\theta)$
- First-order condition: ____________
- $b = __________________________$
- Even with risk-neutral agent, $b < 1$
- Loss due to gaming:
  - $E[e] = E[bk] = b < 1 = e^*$

When the agent can manipulate the performance measure, it is not optimal to tie too much of compensation to this dysfunctional measure!
Contract Based on Subjective Measure q

- $w = a + bq$

- $E[V] = E[q] - E[w]$
  - $= e - 0.5e^2$ (PC) since $E[w] = c(e)$
  - $= b - 0.5b^2$ (IC) since $e = b$

- First-order condition: _______________

  $\Rightarrow b = ______________$

  $\Rightarrow E[e] = ______________$
Comparison

- With objective performance measure (p)
  - $e = \frac{k}{1+\theta}$
  - $E[V] = E[e - 0.5e^2]
    = \frac{1}{1+\theta} - 0.5E[k^2]/(1+\theta)^2$
    $= 0.5/(1+\theta)$

- With subjective performance measure (q)
  - $e = 1 > \frac{1}{1+\theta}$
  - $E[V] = e - 0.5e^2 = 0.5 > 0.5/(1+\theta)$

Why not always use q?
Trust

- Using subjective performance measures may induce efficient outcomes and eliminate the need for using dysfunctional objective measures.

- However, the agent must believe that the principal will honor his promise to pay based on $q$.
  - Not possible to enforce the contract in courts since $q$ cannot be verified.
  - Only self-enforcing contracts possible.
One-Period Interaction

- Suppose $q=1$

- Principal’s payoffs:
  - Claim $q=1$ $\Rightarrow V = q-w=0.5$
  - Claim $q=0$ $\Rightarrow V = q=1$

$\Rightarrow$ Clear incentives to renege on promise (lie)!
Infinite Period Interaction

- **Trigger strategy**
  - Co-operate unless one party defects, in case of which revert to contract with objective measure p forever

- **Principal’s payoffs**:
  - Claim \( q = 1 \):
    \[
    0.5 + \beta 0.5 + \beta^2 0.5 + \ldots = \text{__________________________}
    \]
  - Claim \( q = 0 \):
    \[
    1 + \frac{\beta 0.5}{1 + \theta} + \frac{\beta^2 0.5}{(1 + \theta)} + \ldots = \text{__________________________}
    \]
  - where \( \beta \) is the discount factor
Conditions for Using Subjective Measures

- \( D = V(\text{claim } q=1) - V(\text{claim } q=0) \)
  \[ = \{0.5 + 0.5 \frac{\beta}{1-\beta}\} - \{1 + [0.5/(1+\theta)] \frac{\beta}{1-\beta}\} \]

- \( D = -0.5 + \left(0.5 - \frac{0.5}{1+\theta}\right) \left(\frac{\beta}{1-\beta}\right) \)

- Two main conditions for using subjective signals:
  1. Large \( \beta \) (i.e. future matters sufficiently to the principal)
  2. Large \( \theta \) (i.e. objective measures sufficiently dysfunctional)
Summary on Gaming and Subjective Evaluation

1. Often, the real outcome that the principal cares about cannot be verified and therefore contracts based on this outcome cannot be enforced in courts.

2. When the agent can manipulate the verifiable performance measures, it is optimal not to tie too much of the agent’s pay to these measures. This in general causes efficiency loss.

3. This efficiency loss can be eliminated if the principal can design the contract based on the unverifiable real outcome that he cares about.

4. The contract based on the unverifiable outcomes must be self-enforcing. This will in general be the case when the principal cares about future interactions with the agent(s) or when the verifiable performance measures are sufficiently corrupt.
Who’s the principal and the agent here???
Biased Reporting

- Many principals and agents
- Some principals (e.g. supervisors) not residual claimants; therefore, don’t gain by cheating/reneging on promise

- Reporting can still be **biased** for other reasons

Centrality Bias

\[ \text{Var}[q^R] < \text{Var}[q^A] \]

Leniency Bias

\[ \text{E}[q^R] > \text{E}[q^A] \]
Why is there bias?

1. Information Gathering
   - Costly monitoring with no incentives to do it right
   - ‘Halo’ effect: global impression, prior performance, ...

2. Negative Consequences
   - Negative feedback unpleasant, damages personal relationships, leads to discussion and criticism, etc.

3. Cognitive Limitations
   - Lack of experience and knowledge of outcomes
Why is there bias?

4. Favoritism

- Personal considerations:
  - Reward appreciated employees
  - Encourage loyalty
  - Promote personal agendas
  - Punish rebellious employees

Problem: Rent-Seeking Activities

- Agent can improve pay in two ways:
  1. Work hard (production)
  2. Influence principal’s evaluation (rent-seeking)
Preventing Biased Ratings

1. Forced Ratings
   - Categorize workers into groups
   - Ensures that distribution of outcomes unbiased
   - Problems:
     - Employees compete against each other, which may induce sabotage and discourage co-operation
     - Performance based on other agents’ performance, which introduces additional risk
     - Does not address favoritism

Multilayer Organizations
2. Punish biased ratings
   - Monitoring very costly
   - Allow agents to appeal:
     - Ineffective because of under-reporting (retaliation) and because management is reluctant to reverse decisions

3. Training
   - May make principals aware of their unconscious biases and cognitive limitations
   - Does not address biasing rating deliberately
Preventing Biased Rating

4. Bureaucratic Rules

- Example: promotion based on seniority
- Advantage: less corruptible
- Disadvantage: not necessarily related to performance
Application: Evaluation of Students

- Forced rating: A (5-25%), A+B (<67%), F<50%
- Punish biased rating
  - Evaluation of appeals by an independent party
- Continuing education on best practices in evaluation
- Bureaucratic rule: small weight on class attendance
- Preventing favoritism:
  - Mask student names and numbers
  - Keep the identity of marker (TA) confidential
Summary on Multilayer Organizations

1. In some relationships, such as those in the multilayer organizations, the principal is not the residual claimant.
2. Even though in this case the principal does not directly benefit for misreporting on the agent’s performance that cannot be verified, the evaluation may still be bias for other reasons.
3. These reasons include: costly information gathering, cognitive limitation, negative personal consequences, and favouritism.
4. Biased rating can be prevented using several methods, such as: punishing biased rating, forced ratings, training, and bureaucratic rules.
1. No Uncertainty
2. Uncertainty

3. Risk-Averse Agent
4. Risk-Neutral Agent
5. Multiple Signals
6. Multiple Tasks
7. Subjective Signals
8. Intrinsic Motivation

9. Teams
10. Tournaments
Proof that $E[k^2] = \theta - [E(k)]^2$

- $\theta = E[k - E(k)]^2$
  
  $= E[k^2 + E(k)^2 - 2kE(k)]$
  
  $= E[k^2] + [E(k)]^2 - 2E(k)E(k)$
  
  $= E[k^2] - [E(k)]^2$

- When $E(k) = 1$, we have that $[E(k)]^2 = 1$, and

  $\theta = E[k^2] - 1$
  
  $\Rightarrow E[k^2] = 1 + \theta$
Sum of infinite series

- Consider an infinite series \( s = \text{constant} \times (1 + \beta + \beta^2 + \ldots) \)
- Then, \( \beta s = \text{constant} \times (\beta + \beta^2 + \beta^3 + \ldots) \)
- Then, \( s - \beta s = \text{constant} \), and

\[
s = \text{constant} / (1 - \beta) \\
\beta s = \text{constant} \times [\beta / (1 - \beta)]
\]