

# Risk Neutral Agent

Class 4

# How to Pay Tree Planters?



## Consequences of Hidden Action

- $q=e+u$
- $u \sim (0,\theta)$
- $c(e)=0.5e^2$
- Agent is risk averse
- Principal is risk neutral
- $w = a + bq$

No Hidden  
Action

$$b^* = 0$$
$$e^* = 1$$

- Optimal Risk Insurance
- Optimal Incentives

Hidden  
Action

$$b = 1/(1+r\theta) > 0$$
$$e = 1/(1+r\theta) < 1$$

- Inefficiently high  $b$
- Inefficiently low  $e$

# Addressing Efficiency Loss

- With hidden action, there is an efficiency loss because

$$b = 1/(1+r\theta) = e < e^* = 1$$

- At least two ways to improve incentives (i.e. increase b):

1. Contract with less risk-averse agent (lower  $r$ )
2. Reduce uncertainty (lower  $\theta$ )

# Objectives for Today

1. Optimal Contract with Risk Neutral Agent
2. Review: Regression Analysis
3. Empirical Methods: Randomized Experiments
4. Application: Tree planting Contracts in B.C.

Element	Description
Parties	Principal, Agent
Production Technology	$q=e+u$ , where $u \sim (0, \theta)$
Information	$e$ <b>cannot</b> be observed or verified
Contract	$(a, b)$ where $w=a+bq$
Payoffs	Agent: $U=u(w)-c(e)$ , with $r=0$ and $c(e)=0.5e^2$ Principal: $V=V(q-w)$ , with $s=0$
Outside Options	$R=0=S$
Timing	<ol style="list-style-type: none"> <li>1. P design contract <math>(a,b)</math></li> <li>2. A accepts or rejects</li> <li>3. If A accepts, A chooses <math>e</math></li> <li>4. Production and payoffs</li> </ol>



# The Contract Design Problem

- Max<sub>a,b</sub> E[V]  
s.t. (PC) E[U] ≥ 0  
(IC) e maximizes E[U]

- $E[V] = E[q-w] - 0.5s\text{Var}[q-w]$

$$= \underline{\hspace{15cm}}$$

- $E[U] = E[w] - 0.5r\text{Var}[w] - c(e)$

$$= \underline{\hspace{15cm}}$$

# Solution by Backward Induction

## 1. Incentive compatibility constraint (ICC)

- $\text{Max}_e E[U] = a + be - 0.5e^2$
- (IC) \_\_\_\_\_

## 2. Participation constraint (PC)

- $E[U] = a + be - 0.5e^2 = 0$
- ⇒ (PC)  $a =$  \_\_\_\_\_



# Optimal Contract

## 3. Max principal's objective, subject to ICC and PC

$$\begin{aligned}
 \circ \quad E[V] &= (1-b)e^{-a} \\
 &= (1-b)e^{-(0.5e^2 - be)} && \text{from (PC)} \\
 &= (e - 0.5e^2) \\
 &= (b - 0.5b^2) && \text{from (ICC)}
 \end{aligned}$$

- Max  $E[V] = (b - 0.5b^2)$
  - First-order Condition:
-

# Interpretation

- $b^*=1 \Rightarrow e=1=e^*$ 
  - Even with hidden action, the principal can induce the optimal action!
  - Intuition:  $b$  only has to provide incentives since the agent is risk-neutral (no insurance problem).
  
- $a^*=c(e^*)-q(e^*) \Rightarrow a^* \leq 0$ 
  - Social Surplus =  $E[V]+E[U]$   
 =  $q(e)-c(e)$   
 =  $-a \geq 0$  to form a relationship



	Taxi Drivers	Physicians	Franchises
Fixed Pay	Rent	Medical License	Purchasing fee
Variable Pay	Entire Fares and Tips	Full value of services provided to patients	Entire Profits

In all of these examples:

- The principal sells the 'job' to the agent for a fixed amount
- The agent keeps entire output

This contract works because:

- The agent is the residual claimant → no incentive to 'shirk'
- The agent is indifferent to absorbing all risk

# Importance of Testing Theories

- “Personnel Economics: Too Many Theories, Too Few Facts!”  
*G. Baker and B. Holmstorm (1995)*
- “Errors using inadequate data are much less than those using no data at all.”  
*C. Babbage*
- “First, get the facts, then you can distort them at your leisure.”  
*M. Twain*



# Theoretical Predictions

- Suppose:
  - $q=e+u$
  - $c(e)=0.5e^2$
  - Effort cannot be observed
  - Piece rate contract  $a+bq$
  
- From (ICC)  $e=b$ :
  - Salary ( $b=0$ )       $\Rightarrow e=$  \_\_\_\_\_  $\Rightarrow E[q]=$  \_\_\_\_\_
  - Piece rate ( $b=1$ )    $\Rightarrow e=$  \_\_\_\_\_  $\Rightarrow E[q]=$  \_\_\_\_\_

# Regression Model

- Used to study a relationship between an outcome variable and one or more explanatory variables
- Example

$$y_i = \alpha + \beta D_i + \varepsilon_i$$

where:

- $i$  indexes workers,  $i=1,2,\dots,n$
- $y$  = output
- $D$  = indicator, equal to 1 if piece-rate, 0 if salary
- $\varepsilon$  = all other variables that affect output

# Estimation

Worker (i)	Productivity (Y)	Piece Rate (D=1 if yes)
1	4	1
2	2	0
3	3	0
4	5	1
5	3	1
6	2	0
7	4	1

$$Y_i = \alpha + \beta D_i + \varepsilon_i$$

- The goal is to estimate coefficients  $\alpha$  and  $\beta$  that 'best fit the data'
- Common method is the least squares:

$$\text{Min}_{\alpha, \beta} \sum_j (Y_i - \alpha - \beta D_i)^2$$



# Regression Output

Variable	Coefficient	Standard Error	t-statistic
Piece Rate ( $\beta$ )	1.67	0.56	2.99
Constant ( $\alpha$ )	2.33	0.42	5.53

$$E[Y_i] = 2.33 + 1.67D_i$$

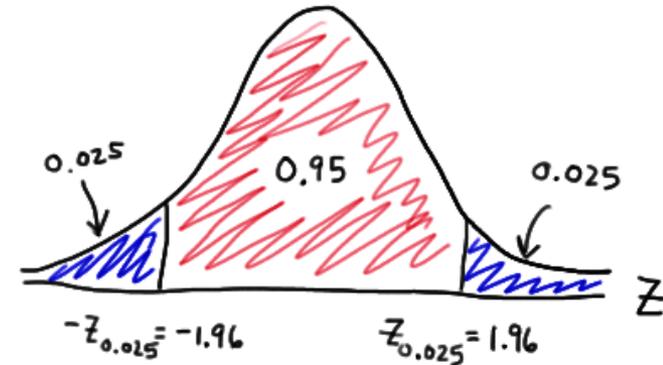
Salary :  $E[Y | D=0] = \underline{\hspace{2cm}}$

Piece Rate :  $E[Y | D=1] = \underline{\hspace{2cm}}$

# Three Main Questions

## 1. Does the relationship exist?

- Statistical significance
- $t\text{-statistic} = \text{coefficient} / \text{standard error}$
- Tests whether the coefficient is zero
- $\text{Absolute}(t\text{-statistic}) > 2 \Rightarrow \text{significant}$



## 2. Is the relationship positive or negative?

- Sign of coefficient



## 3. How strong is the relationship?

- Economic significance
- Magnitude of coefficients



# Application

Variable	Coefficient	Standard Error	t-statistic
Piece Rate	1.67	0.56	2.99
Constant	2.33	0.42	5.53

- The relationship between output and type of compensation is positive (+1.67) and statistically significant ( $t=2.99$ ). Specifically, the piece-rate workers produce on average 1.67 units more than salary workers, all else equal. This impact is economically significant, representing a gain in productivity of about 72% (i.e. 1.67 units gain relative to 2.33 for salary workers).



# Treatment and Selection Effects

- $Y_i = \alpha + \beta D_i + \varepsilon_i$
- Suppose you got average  $Y$  in the group with  $D=0$  and in the group with  $D=1$ . What does the difference in these averages represent?
- $E[y | D=1] - E[y | D=0]$   
 $= \alpha + \beta + E[\varepsilon | D=1] - (\alpha + E[\varepsilon | D=0])$   
 $= \underline{\hspace{4cm}}$
- $\beta =$  treatment effect  
 (difference in productivity due to payment method only)
- $E[\varepsilon | D=1] - E[\varepsilon | D=0] =$  selection effect  
 (difference in all other factors that impact productivity)



# Example

	Output if D=0	Output if D=1
Control Group (Salary workers, D=0)	5	
Treatment Group (Piece-rate workers, D=1)		10

- We observe: 5 for salary workers, 10 for piece rate workers.
  - The observed difference is \_\_\_\_\_.
  - Treatment effect = \_\_\_\_\_.
  - Selection effect = \_\_\_\_\_.
- Correlation  $\neq$  Causation

# Identification Problem

- Treatment and control groups may have different outcomes even in the absence of treatment:

$$E[\varepsilon | D=1] \neq E[\varepsilon | D=0]$$

Identification Problem:  
How to make control and treatment groups  
comparable?





# Controlling for Confounding Factors

- When differences between groups can be observed  
⇒ Estimate the impact of treatment, conditional on the observable differences
- Example:  $Y_i = \alpha + \beta D_i + \gamma X_i + \varepsilon_i$ 
  - where  $X_i$  = college education of worker  $i$
- Identification Assumption

Workers of same education level are identical in everything except how they are paid.

# Example

Worker (i)	Productivity (Y)	Piece Rate? (D)	College (X)
1	4	1	1
2	2	0	1
3	3	0	0
4	5	1	1
5	3	1	0
6	2	0	0
7	4	1	1

$$E[Y_i] = 2.33 + 1.67D_i$$

$$E[Y_i] = 2.18 + 1.47D_i + 0.47X$$

Important to compare workers with same level of education! The impact of piece rate (1.47) smaller than when not comparing workers with same level of education (1.67).

# Unobservable Differences

- Sometimes differences are not observable by researcher (e.g. ability) → Need alternative empirical strategies to try to make  $E[\varepsilon | D=1] = E[\varepsilon | D=0]$
- Empirical Strategies in This Course:
  1. Randomized Experiments
  2. Difference-in-Differences

# Randomized Experiments

- The most credible empirical strategy
- Individuals randomly assigned to treatment and control groups (e.g. by flipping a coin)
- Random assignment ensures that:

$$E[\varepsilon | D=1] = E[\varepsilon | D=0]$$

- Treatment and control groups similar in all aspects except for treatment





# Example

	Salary	Piece Rate
High Ability	20	25
Low Ability	5	10

- Randomize individuals into salary and piece rate firms
- $E[y \mid \text{Salary}] = \underline{\hspace{4cm}}$
- $E[y \mid \text{Piece Rate}] = \underline{\hspace{4cm}}$
- $E[y \mid \text{Piece Rate}] - E[y \mid \text{Salary}] = \underline{\hspace{4cm}}$

# Summary of Empirical Methods

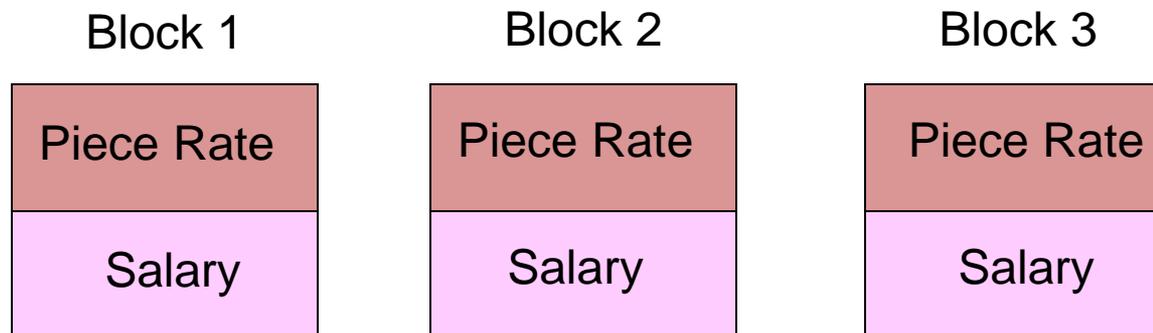
Empirical Method	Representation	Identifying Assumption
Bivariate regression	$y = \alpha + \beta D + \epsilon$	Similar in everything except D
Multivariate regression	$y = \alpha + \beta D + \gamma X + \epsilon$	Similar in everything except D and X
Randomized experiment	$E[y D = 1] - E[y D = 0]$	Randomization ensures similarity in everything except D.



# Application: Shearer (2002)

## Experimental Design

- Nine workers randomly selected from the firm
- Three plots of land, different in terms of planting conditions
- Each plot divided into two parts: piece rate pay and salary
- At the beginning of a day, each worker randomly assigned to one plot and type of pay



# Controlling for Confounding Factors

- Observable differences between piece rate and salary workers that may affect their productivity:
  - Workers not informed about experiment
  - Male workers only
  - Length of work day constant
  - Blocks of land large enough
  - Same supervisor for all workers

# Experimental Results

	Obs.	Piece Rate (Mean Trees)	Salary (Mean Trees)	Percent Difference
All Plots	120	1,256	1,037	21.1%
Plot 1	24	1,390	1,157	19.3%
Plot 2	48	1,501	1,142	31.4%
Plot 3	48	944	873	8.1%

➤ Piece rate workers on average about 20% more productive than salary workers.

# Limitations of Randomized Experiments

## 1. Sometimes Difficult or Impossible to Implement

- Monetary Costs
- Ethical and Legal Considerations
  - Randomize people into smoking (treatment) and non-smoking (control) groups to study impact of smoking on health
  - Randomly annul marriages to assign children to either married parents group (treatment) or single parent group (control) to study impact of divorce on children's psychological development



## 2. External Validity

- Do the results generalize from the unique and idiosyncratic settings, procedures and participants to other populations and conditions?

# Main Points

1. **Risk Neutral Agent**: When the agent is risk neutral, the moral hazard is not a problem because the principal can sell the 'job' to the agent and let the agent keep all output.
2. **Treatment and Selection Effects**: Testing whether one payment method improves productivity compared to another methods faces the challenge of separating the observed difference in productivity into the effect that can be attributed solely to the payment method (treatment effect) and the effect of all other differences between workers in different payment models (selection effect).
3. **Randomized Experiments**: Randomization of workers into different payment methods ensures that the workers are similar in all other aspects. The treatment effect can then be obtained by simply comparing the average difference in productivity between workers in the different payment models.